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PROBLEMS.

67. Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics in Drury College, Springfield, Missouri.

A agreed to work a year for \$300 and a suit of clothes. At the end of five months he left, receiving for his wages \$60 and the clothes. What was the suit worth?

68. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

The population of a city is annually increasing $m=2\frac{1}{2}\%$. If the population now is $P=68921$, what was it $n=3$ years ago? At this rate of increase, what will the population be $n=3$ years hence?

GEOMETRY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

58. Proposed by I. J. SCHWATT, Ph. D., Instructor in Mathematics, University of Pennsylvania, Philadelphia, Pennsylvania.

1. The point of intersection K_a' of the tangent drawn to the circumcircle about the triangle ABC at A and the side BC is harmonic conjugate to K_a with respect to BC. (K_a is the point where the symmedian line through A of the triangle ABC meets the side BC.)

2. The point K_a' is the center of the Apollonius circle passing through A of the triangle ABC.

3. Grebes point is on the line joining the middle point of any side of a triangle with the middle point of the altitude to this side.

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

1. In trilinears, the equation to the circumcircle of the triangle of reference is

$$a\beta\gamma+b\alpha\gamma+c\alpha\beta=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

The tangent to this circle at A is

$$b\gamma+c\beta=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2).$$

The equation to the symmedian through A is

$$b\gamma-c\beta=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3).$$

(2) and (3) are conjugates to $\beta=0$, $\gamma=0$.

2. The circle of Apollonius passes through A and the points of intersection of the internal and external bisectors of the angle A of the triangle of reference with the side BC. The coordinates of the center of this circle are plainly the half sum of those of the intersections of the bisectors with BC, or $\left(0, \frac{ab^2\sin C}{b^2-c^2}, \frac{ac^2\sin B}{b^2-c^2}\right)$. This is the point of intersection of (2) and $\alpha=0$.

3. The coordinates of Grebe's point are proportional to a, b, c.

The mid-point of a is $(0, \frac{a}{2}\sin C, \frac{a}{2}\sin B)$, and of the altitude on a, $\left(\frac{\triangle}{a}, \frac{b}{4}\sin 2C, \frac{c}{4}\sin 2B\right)$.

The line through these points is $\alpha\sin(B-C)+\gamma\sin C-\beta\sin B=0$, which is satisfied by $\alpha=a$, $\beta=b$, $\gamma=c$.

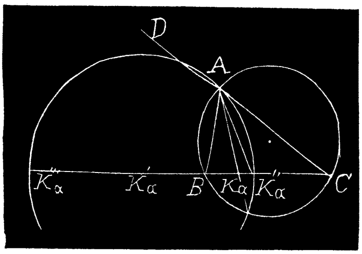
II. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

Using trilinear coordinates we get,

(1), equation to AK_a' is, $\beta\sin C+\gamma\sin B=0$, or $\beta c+\gamma b=0$; to AB, $\gamma=0$; to AK_a, $\beta c-\gamma b=0$; to AC, $\beta=0$.

$\therefore AK_a'$, AB, AK_a, AC form a harmonic pencil.

$\therefore K_a'$ and K_a are harmonic conjugate.



(2). Draw AK_a''' bisecting $\angle DAB$.

Then $\angle AK_a'B=\angle K_a'K_a'''A+\angle K_a'AK_a'''=B-C$, and

$\angle K_a'AK_a'''=\angle K_a'''AB-\angle K_a'AB=90^\circ-\frac{1}{2}A-C=\frac{1}{2}(B-C)$.

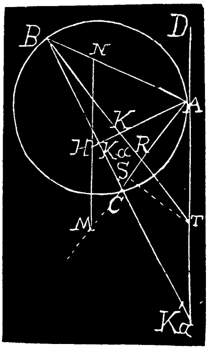
$\therefore \angle K_a'K_a'''A=\angle K_a'AK_a'''$; $\therefore K_a'K_a'''=AK_a'=AK_a''$.

$\therefore K_a'$ is the center of the required circle.

(3). The equation to the straight line through Grebe's point and the mid-point of BC is the same as the equation of the straight line through mid-point of BC and the midpoint of the perpendicular from A on BC, both being $\sin(B-C)\alpha-\sin B\beta+\sin C\gamma=0$.

III. Solution by F. M. McGAW, A. M., Professor of Mathematics, Bordentown Military Institute, Bordentown, New Jersey, and J. C. CORBIN, Pine Bluff, Arkansas.

(1). Let ABC be $\triangle$; AK_a', the tangent at A; and K_a point where symmedian meets the side BC. Take $AN=AC$, and $AM=AB$. Then in $\triangle ANM$, AH, the median, is the symmedian of $\triangle ABC$. Lines BC, and MN are antiparallel. Also, since $\sphericalangle ABC=\sphericalangle BAD$, each being measured by ½arc AC, the lines BC and AK_a' are antiparallel. Wherefore MN is parallel to the tangent line AK_a'.



Now we have a pencil of four rays AB, AH, AC, AK_a' in which one ray AH bisects a line parallel to its conjugate, and included between the other pair of conjugate rays; hence the pencil is harmonic, and any line, as BK_a', drawn across the pencil will cut out an harmonic range $\{BC, K_aK_a'\}$. Q. E. D.

(3). Draw ST perpendicular to AC at its middle point S; draw BT, and it is a symmedian line (Halsted: Syn. Geom. §648.), hence it passes through Grebe's point (or Lemoine's Point) K. Now as $A\{BC, K_aK_a'\}$ is an harmonic pencil, $\{BR, KT\}$ is an harmonic range; whence $S\{BR, KT\}$ is an harmonic pencil. Draw altitude BP, and it is || to ray ST, and is therefore bisected by the ray SK, the conjugate of ST. Therefore the line joining the middle point, S, of a side, and the middle point of the altitude to that side passes through Grebe's point.

Q. E. D.

59. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Show that the tangent plane at any point of the surface $a^2x^2+b^2y^2+c^2z^2=2bcyz+2acxz+2abxy$ intersects the surface $ayz+bzx+cxy=0$ in two straight lines at right angles to one another.

Solution by the PROPOSER.

The tangent plane at (x', y', z') to $F=0$..........................(1) is

$$(x-x')\frac{dF}{dx'}+(y-y')\frac{dF}{dy'}+(z-z')\frac{dF}{dz'}=0 \quad\text{..........................(2).}$$

Here $F=a^2x^2+b^2y^2+c^2z^2-2bcyz-2acxz-2abxy$..................(3).

$$\frac{dF}{dx'}=2a(ax'-by'-cz'),\ \frac{dF}{dy'}=2b(-ax'+by'-cz'),$$

$$\frac{dF}{dz'}=2c(-ax'-by'+cz') \quad\text{......................(4).}$$

Then (2) becomes by aid of (3),

$$a(ax'-by'-cz')x+b(-ax'+by'-cz')y+c(-ax'-by'+cz')z=0 \quad\text{.............(5).}$$